# The Elasticity of Substitution between Capital and Labor in the U.S. Economy

## A Bayesian Meta-Regression Analysis

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# Motivation

- EOS is (implicitly) part of major public discussions about **taxation** and questions regarding the **income and wealth distribution**, or the future of **economic growth**
- Not only textbook models rely on the assumption of a Cobb-Douglas production function for the sake of convenience ...
- ... but also because it seems to be a reasonable assumption
  - Factor shares remain fairly stable over time
  - Influential authors suggested a unitary elasticity has empirical backing, i.e. Berndt (1976)

# Motivation

- Theory just tell us $\sigma$ is supposed to be non-negative

$$Y = A^H \left[\pi(A^K K)^{\frac{\sigma-1}{\sigma}} + (1-\pi)(A^L L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma = [0, \infty)$$

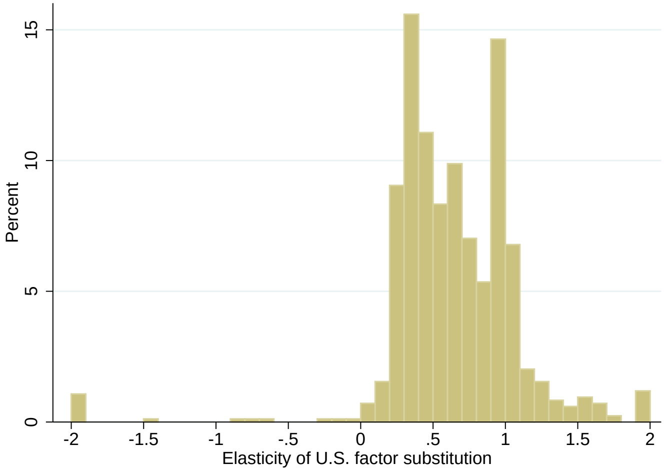
$$\sigma = \frac{\% \text{ change in capital intensity}}{\% \text{ change in relative factor prices}}$$

$\sigma = 0$ ... Walras-Leontief

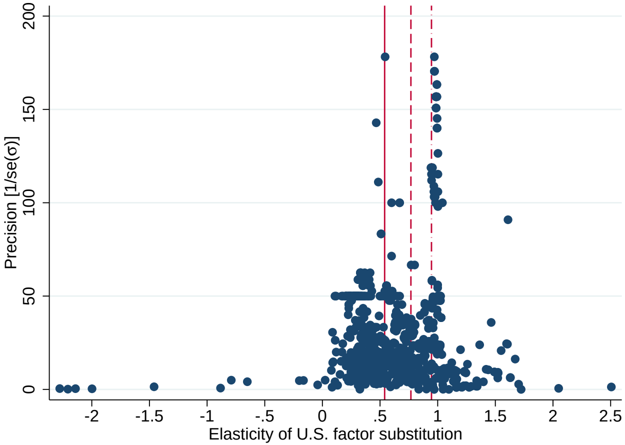
$\sigma = 1$ ... Cobb-Douglas

$\sigma \to \infty$ ... von Neumann

# Motivation


15
10
5
0
Percent
-2
-1.5
-1
-.5
0
.5
1
1.5
2
Elasticity of U.S. factor substitution

# Funnel Plots of $\tilde{\sigma}$




$-3 < \tilde{\sigma} < 3$ & $1/se(\tilde{\sigma}) < 200$; solid line: mean; dashed line: $1/se(\tilde{\sigma})$ weighted mean; dash-dotted line: $1/se(\tilde{\sigma})^2$ weighted mean

# Bayesian Meta-Regression

- Research Questions:
  1. What causes heterogeneity in estimates of $\sigma$?
  2. What is the “benchmark” U.S.-$\sigma$?
- Possible sources of heterogeneity:
  - estimation equations
  - estimation techniques
  - types of data and data configuration
  - short-run/long-run estimation
  - ...
- Sample: 48 studies, 840 observations
- Method: Bayesian linear mixed effects model

# Why Bayesian?

- Makes use of **prior knowledge** within the regression framework.
- Can "test" whether our data are strong enough to **convince researchers with different prior beliefs** about $\sigma$ to agree on a consensus range
- Can estimate a **posterior density** of $\sigma$: Given the data (properties of previous studies) and a defined empirical best practice approach, how likely is $\sigma$ to fall into the range of ...

# Bayesian Meta-Regression

For each categorical variable, we choose a "best practice" as reference category

- An estimate of the distribution of the value of U.S.-$\sigma$ is given by the posterior distribution of the constant term
- $\hat{\sigma} = 0.59 + 0.54 \times D_{noTechdyn}$

How to determine the "benchmark"?

- evidence provided by Monte-Carlo simulation Leon-Ledesma et al. (2010)
- handling of estimation problems / restrictions (endogeneity, ...)
- theoretical or empirically guided reasoning

# Bayesian Meta-Regression

Table: Reference categories that represent a "best-practice" estimation

| Reference category | Rationale |
|---|---|
| ***Study characteristics*** | |
| Journal article estimates | Review process ensures higher quality |
| ***Estimation equation*** | |
| Equation system | Evidence from Monte Carlo simulations |
| Mark-up | Accounts for imperfect (factor) markets |
| Estimations in levels | Avoids approximation error |
| ***Technological dynamics*** | |
| Factor biased, Box-Cox | Allows for non-linear dynamics |
| ***Estimation*** | |
| IV estimations | Accounts for endogeneity issues |
| ***Data characteristics*** | |
| Country | Captures economy-wide substitution possibilities |
| Time series | Ensures consistency with "Country" |
| Quality adjusted labor | Accounts for skill differences |

# Bayesian Meta-Regression I

- Three combinations of priors are utilized:
  (1) "Non-informative"
  (2) "Preferred Specification"
  (3) "Cobb-Douglas"
- "Normal" priors on the coefficients, except for the constant

(1) Represents a researcher knowing only the theoretical concept of the elasticity of substitution:
  - "flat" IG(0.1,1)-prior on $\sigma$
  - All other coefficient priors are set to N(0,100)

# Bayesian Meta-Regression II

- Three combinations of priors are utilized:
  (1) "Non-informative"
  (2) "Preferred Specification"
  (3) "Cobb-Douglas"
- "Normal" priors on the coefficients, except for the constant

(2) Represents a researcher with prior knowledge of previous empirical and theoretical studies.

- Priors make use of evidence from Monte Carlo simulations
- $\sigma \sim$ Rayleigh(0.48) with $E[\sigma] = 0.6$

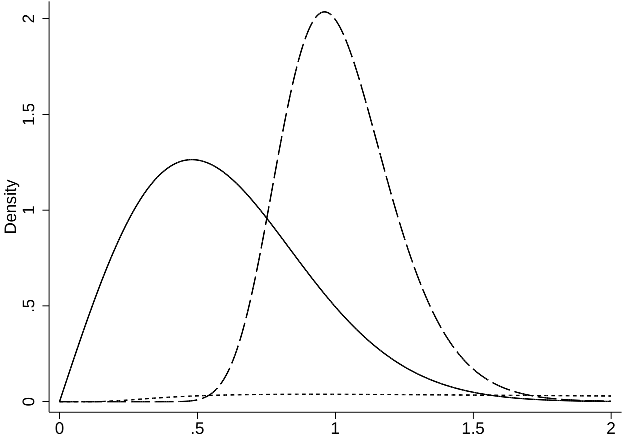
# Bayesian Meta-Regression III

- Three combinations of priors are utilized:
  (1) "Non-informative"
  (2) "Preferred Specification"
  (3) "Cobb-Douglas"
- "Normal" priors on the coefficients, except for the constant

(3) Represents a researcher with knowledge as in (2) but
  - believing $\sigma$ to be 1: $\sigma \sim \text{LN}(0, 0.2)$
  - Same coefficient priors as (2)

# Bayesian Meta-Regression

Prior specifications on $\sigma_0$




dashed: Rayleigh(0.48), solid: LN(0,0.2), dotted: IG(0.1,1)

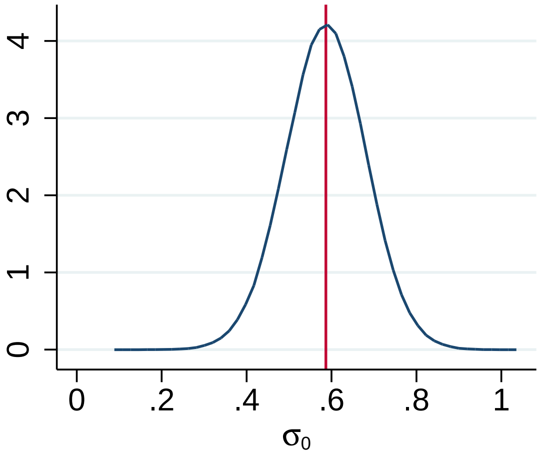
# Results (Selection)

Results of Bayesian linear mixed effects model estimations using inverse standard error weighting

| | Estimate of $\sigma_0$ | | | | |
|---|---|---|---|---|---|
| | | CRI (LB) | | CRI (UB) | |
| | **Mean** | 90% | 80% | 80% | 90% |
| Preferred prior | **0.59** | 0.43 | 0.47 | 0.71 | 0.74 |
| Cobb-Douglas prior | **0.71** | 0.58 | 0.61 | 0.8 | 0.83 |
| Non-informative prior | **0.57** | 0.41 | 0.44 | 0.7 | 0.74 |

# Bayesian Meta-Regression

Posterior density of $\sigma_0$ for the "preferred prior"



# Results (Selection)

$$Y = A^H \left[ \pi (A^K K)^{\frac{\sigma-1}{\sigma}} + (1-\pi)(A^L L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Technological dynamics:

- factor biased, Box-Cox (**reference**)
- factor biased, linear
- factor biased, other
- Hicks neutral
- no technological dynamics

# Results (Selection)

Results of Bayesian linear mixed effects model estimations using inverse standard error weighting

| | | Preferred prior | | | |
|---|---|---|---|---|---|
| | | CRI (LB) | | CRI (UB) | |
| Variable | **Mean** | 90% | 80% | 80% | 90% |
| Factor biased, Box-Cox (Ref.) | **-** | - | - | - | - |
| Factor biased, constant growth | **0.38** | 0.34 | 0.35 | 0.41 | 0.42 |
| Factor biased, other | **0.16** | -0.26 | -0.16 | 0.48 | 0.58 |
| Hicks neutral, constant growth | **0.38** | 0.33 | 0.34 | 0.43 | 0.44 |
| No dynamics | **0.55** | 0.46 | 0.48 | 0.62 | 0.64 |

# Results (Selection)

Results of Bayesian linear mixed effects model estimations using inverse standard error weighting

| | Cobb-Douglas prior | | | | |
|---|---|---|---|---|---|
| | | CRI (LB) | | CRI (UB) | |
| Variable | **Mean** | 90% | 80% | 80% | 90% |
| Factor biased, Box-Cox (Ref.) | **-** | - | - | - | - |
| Factor biased, constant growth | **0.37** | 0.32 | 0.33 | 0.40 | 0.41 |
| Factor biased, other | **0.20** | -0.21 | -0.12 | 0.52 | 0.62 |
| Hicks neutral, constant growth | **0.37** | 0.32 | 0.33 | 0.41 | 0.42 |
| No dynamics | **0.53** | 0.44 | 0.46 | 0.60 | 0.62 |

# Results (Selection)

Results of Bayesian linear mixed effects model estimations using inverse standard error weighting

| | Non-informative prior | | | | |
|---|---|---|---|---|---|
| | | CRI (LB) | | CRI (UB) | |
| Variable | **Mean** | 90% | 80% | 80% | 90% |
| Factor biased, Box-Cox (Ref.) | **-** | - | - | - | - |
| Factor biased, constant growth | **0.38** | 0.33 | 0.34 | 0.41 | 0.42 |
| Factor biased, other | **0.13** | -0.31 | -0.21 | 0.46 | 0.56 |
| Hicks neutral, constant growth | **0.38** | 0.33 | 0.34 | 0.42 | 0.44 |
| No dynamics | **0.54** | 0.45 | 0.47 | 0.61 | 0.63 |

# Results (Selection)

Further results (first is reference):

- estimation equation (equation system, FOC, linear approx., production function)
- data type (time series, panel, cross-section)
- data level (national, industry, firm level)
- human capital corrected / unadjusted labor
- long run $\sigma$ / short run / theor. long-run, emp. short-run
- publication type (journal, working paper, monograph)
- consideration of mark ups / no mark-ups
- estimation in levels / growth rates
- IV estimation / no IV
- estimation method (least squares, other)

# Sensitivity and Extensions

- Results are fairly robust across prior assumptions
- Variance weighting does not change the results qualitatively, but is not as conclusive
- We find small time effects. Starting from a Leontief production function, the U.S. would approach Cobb-Douglas within 330 years

# Conclusion

1. What causes heterogeneity in estimates of $\sigma$?
   - Far most important factor is the operationalization of technological dynamics
   - Not estimating systems yields smaller estimates of $\sigma$
   - Monographs publish more and more controversial estimates
2. What is the "benchmark" U.S.-$\sigma$?
   - We belief $\sigma \in [0.5, 0.7]$
   - Cobb-Douglas is always rejected!